2020
MATHEMATICS
[HONOURS]
Paper : VII

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Give classical definition of probability.

b) For any event 'A' connected with the random experiment E with event space 'S', show that $P(\overline{A})=1-P(A)$ where $\overline{A}$ is the complement of A.

c) Show that two perfect events A and B cannot be simultaneously mutually exclusive and independent.

d) Define confidence interval.

e) State fundamental theorem of duality.

f) Find the value of the constant k such that the function given by

$$f(x)=Kx(1-x),\ 0<x\le 1$$
$$=0 \qquad \text{elsewhere,}$$

is a possible probability density function of a distribution.

g) Prove that $E(ax+b)=aE(x)+b$, where a, b are constants.

h) Find mode and median of the following data: 9, 2, 8, 7, 1, 1, 4, 5, 6, 6, 2, 9, 7, 8 4, 6.

i) Prove that $\operatorname{Var}(X)=E(X^2)-m^2$, where m = E(X).

j) Find the correlation coefficient of X and Y if $4x+y=52$ and $x+y=32$, be the regression lines of x on y and of y on x respectively.

k) Show that a hyperplane is a convex set.

l) State necessary conditions to form a loop in a transportation problem table.

m) Show that the solution of a transportation problem is never unbounded.

n) Define basic feasible solution of an L.P.P.

o) Define mixed strategy.

p) Find the basic solution of the system

$$x_1 + 2x_3 = 1;\ x_2 + x_3 = 4,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

2. Answer any **three** questions: $8\times3=24$

a) i) State and prove Bayes' theorem on probability.

ii) Find the mean of normal distribution. (1+4)+3

b) i) The random variables X and Y have the joint density function

$$f(x,y) = 6(1-x-y),\ x \geq 0,\ y \geq 0,\ x+y \leq 1$$
$$= 0 \qquad \text{elsewhere}$$

then find the marginal probability density functions of X and Y respectively.

ii) If X and Y be correlated and U and V be defined by

$$U = X\cos\alpha + Y\sin\alpha,\ V = Y\cos\alpha - X\sin\alpha,$$

then U and V will be uncorrelated if

$$\tan 2\alpha = \frac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2},$$

where ρ is the correlation coefficient of X and Y, σ_x^2 and σ_y^2 are the variance of X and Y respectively. 4+4

c) i) X is a discrete random variables having the following probability mass function:

X:	0	1	2	3	4	5	6
P(X=x):	0	k	2k	3k	4k	5k	6k

find k and $P(X \geq 5)$.

ii) Find the variance of Poisson distribution.

iii) A dice is thrown k times in succession. Find the probability of obtaining 'six' at least once. 3+3+2

d) i) State and prove Tchebycheff's inequality.

ii) Prove that for the binomial distrubution, the following recurrence

$$\mu_{r+1} = pq\left(\frac{d\mu r}{dp} + nr\ \mu_{r+1}\right)$$

holds good where μ_r is the rth central moment of the binomial distribution. (1+4)+3.

e) i) Deduce Poisson distribution from binomial distribution stating necessary conditions.

ii) If X has normal (0, 1) distribution, then find the distribution of e^x. 5+3

3. Answer any **two** questions: $8\times2=16$

a) i) Let $X_1, X_2, \ldots X_n$ be a random sample from the population of X and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$ is the sample variance then show that $E(S^2)=\sigma_X^2$, where (σ_X^2) is the population variance and $\bar{X}$ is the sample mean.

ii) The arithmetic mean of a certain distribution is 5. The second and the third moments about the mean are 20 and 140 respectively. Find the third moments of the distribution about 10. 4+4

b) i) Find the maximum likelihood estimator of the parameter μ for a sample of size n from a Poisson population.

ii) Show that the sample mean is an unbiased estimator of the population mean.

iii) Find the first two quartiles from the following data:

Class intervals(Rs.)	5-9	10-14	15-19	20-24	25-29	30-34	35-39
Frequency	10	15	25	40	35	20	5

3+2+3

c) i) Define Chi-square (χ^2) distribution. If X and Y are independent Chi-square distributions with m and n degrees of freedom, then show that X+Y is also a Chi-square distribution with (m+n) degree of freedom.

ii) From a normal population with standard deviation (S.D)3, nine observations are drawn : 25, 20, 18, 27, 23, 32, 19, 21, 22. Obtain the 95% confidence interval for the population mean. 5+3

4. Answer any **five** questions: $8\times5=40$

a) i) Solve the L.P.P. by simplex method

Maximize : $Z = 4x_1+3x_2$

subject to $3x_1+x_2\le15$

$3x_1+4x_2\le24,\ x_1\ge0,\ x_2\ge0.$

ii) Solve graphically the following L.P.P.:

Minimize $Z = 4x_1 - 3x_2$

subject to $2x_1 - x_2 \geq 4$

$4x_1 + 3x_2 \leq 28, \quad x_1 \geq 0, \quad x_2 \geq 0.$

5+3

b) i) If (1, 3, 2) is a feasible solution of the set of equations $2x_1 + 4x_2 - 2x_3 = 10$, $10x_1 + 3x_2 + 7x_3 = 33$ then reduce the above feasible solution to a basic feasible solution.

ii) If an LPP has at least two optimal feasible solution, then show that there are infinite number of optimal solutions which are the convex combination of the initial optimal solutions. 5+3

c) i) Using Big-M method, solve the following L.P.P.:

Maximize $Z = 3x_1 - x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4, \quad x_1 \geq 0, x_2 \geq 0.$

ii) Find the extreme points, if any, of the set $X = \{(x, y), |x| \leq 1, |y| \leq 1\}$. 6+2

d) Solve the following LPP by using two phase method:

Minimize $Z = 4x_1 + x_2$

subject to $x_1 + 2x_2 \leq 3;\ 4x_1 + 3x_2 \geq 6;$ 8

$3x_1 + x_2 = 3;\ x_1 \geq 0, \ x_2 \geq 0.$

e) i) Write down the dual of the following problem and solving the dual problem by simplex method find the optimal values of the primal and dual as well.

Maximize : $Z = 3x_1 + 4x_2$

subject to $x_1 + x_2 \leq 10;\ 2x_1 + 3x_2 \geq 18;$

$x_1 \leq 8;\ x_1 \geq 0, \ x_2 \geq 0.$

ii) How can you convert unbalanced transportation problem into a balanced transportation problem? 6+2

f) i) Find the initial basic feasible solution of the following transportation problem by VAM:

	D_1	D_2	D_3	D_4	a_j
O_1	6	4	2	7	8
O_2	5	1	4	6	14
O_3	6	5	2	5	9
O_4	4	3	2	1	15
b_j	7	13	12	10	

Using UV method test the optimality of the solution.

ii) What is the saddle point of a pay-off matrix in a game? Show that the following pay-off matrix has no saddle point.

		B		
		I	II	III
A	I	4	6	2
	II	1	4	6
	III	3	2	6

3+2+3

g) i) Use simplex method to obtain the inverse of the matrix $\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$.

ii) If a fixed number λ is added to each element of a pay-off matrix, then prove that the optimal value of the game is increased by λ. 6+2

h) i) A salesman has to visit five cities A, B, C, D, E. The distances (in hundred miles) between the five cities are as follows:

	A	B	C	D	E
A	-	6	12	6	4
B	6	-	10	5	4
C	8	7	-	5	3
D	5	4	10	-	5
E	5	2	7	8	-

If the salesman starts from the city A and has come back at A, which route should be selected so that the total distances travelled is minimum and find the minimum distance.

ii) Find the optimal assignment and the optimal assignment cost from the following cost matrix:

	M_1	M_2	M_3	M_4	M_5
J_1	4	6	5	1	2
J_2	7	9	9	6	4
J_3	5	8	5	5	1
J_4	1	3	3	2	1
J_5	6	8	7	6	2

4+4